LIBOR’s Poker *

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Abstract

Inspired by recent scandals of benchmark manipulation, I discuss the survey-based benchmarking as a game of incomplete information in which the principal is unaware of agents’ private signal distributions. I characterize a standard survey in which the expected benchmark bias is distribution-free and corrigeable. If the benchmark is a trimmed average of survey responses, the closed-form equilibrium shows that increasing the panel size and trimming more quotes improves the benchmark accuracy. The expected benchmark bias is not distribution-free under collusion. The result of the standard survey, as an abstraction of actual benchmarking, provides a basis for further research.

Keywords: LIBOR; Interbank lending; Benchmarking; Strategic reporting; Mechanism design

JEL classification: D43, D44, D47, G10, G21

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1 Introduction

A financial benchmark, such as the Chicago Board of Exchange (CBOE) Volatility Index (VIX) or London Interbank Offered Rate (LIBOR), is essentially a statistic of time-varying market attribute. The CBOE VIX, which measures the expected overall stock volatility, is market-based: the liquid market of S&P 500 index options allows CBOE to calculate and disseminate VIX in real-time. Without ample interbank transactions, LIBOR—the robust mean of interbank borrowing costs—is survey-based. The benchmark administrator has to solicit major banks’ best private estimations about their borrowing costs should transactions occur. It is an honor system at heart, since banks know that a thorough verification of their survey responses is cost prohibitive. Recent scandals showed that banks fiddled LIBOR so many times that proved the fiddling was at least widely tolerated, if not unanimously practiced.\(^1\) Nor was the ISDAfix index, another survey-based benchmark for cash settlement of interest rate swaptions, rigging-free.\(^2\) Clearly, the honor system has failed, and the survey is a game of incomplete information in which the benchmark administrator’s task is to discover the statistic of market attributes using participants’ strategic answers.

A standard assumption in the game of incomplete information literature is that the principal knows the distribution of the private type of agents. However, this assumption is inconsistent with the purpose of survey-based benchmarking, which is to find out the average, or other statistical moments, of the private types. Hence, in this study, I investigate the survey as a game of incomplete information where principal does not know the distribution of agents’ private signals. I show that when the agents have symmetric payoff and independent private signals, the principal can implement a quadratic penalty function which leads to an expected benchmark bias that does not vary with the cross-sectional distribution of participants’ private information.

Both LIBOR and ISDAfix affect the value of more than 300 trillion US dollars of interest


\(^{2}\)Developed by International Swaps and Derivatives Association (ISDA), ISDAfix is a global index for cash settlement of interest rate swaptions.
rate swaps, corporate bonds, residential mortgages, and other financial contracts, across mul-
tiple currencies and maturities. By stark contrast, the interbank market is so illiquid that in
each currency and maturity at most a few transactions take place daily. The swap cash
settlement market for ISDAfix is no better. After the LIBOR scandal unfolded, the Interna-
tional Organization of Securities Commissions (IOSCO), the Market Participants Group, and
Duffie and Stein (2015) recommended that key benchmarks fixing—the process of determin-
ing the index—should be “anchored” in actual market transactions or executable quotations.
Nevertheless, the reform of the benchmark has been slow after the emergence of scandals,
as in the follow-up report by IOSCO (2016). LIBOR is still a survey-based benchmark.
Changing a benchmark would mean changing the realms of all contracts and legal documents
that refer to it: thousands of LIBOR-linked loans and ISDAfix-quoted derivative contracts
would have to be redrafted and renegotiated. Therefore, it remains important to study the
survey-based LIBOR and ISDAfix benchmarking. Finally, survey-based benchmarks are used
in over-the-counter markets, and other fields such as industry compensation.

I model a survey as a game of incomplete information between a panel of agents and
a benchmark administrator who surveys the panel’s private signal about the fair obscure
market price and sets the benchmark equal to a linear combination of the order statistics of
the survey results. Agents subject to sparse audit and penalty report strategically.

I first show that the existence of a weakly increasing pure strategy Bayesian Nash equi-
librium under an asymmetric setting with affiliated signals of private information. The equi-
librium is not truthful, and it depends on the cross-sectional distribution of private signals.
In case the administrator knows the distribution of private signals, revelation principle sug-
gests that the administrator may design a direct mechanism to induce truthful reporting.\(^7\)
Unfortunately, assuming the administrator to know the distribution of private signals is in-
consistent with the purpose of financial benchmarking: which is to find out the average, or
other distribution parameters, of market attributes. For example, if the LIBOR benchmark
administrator already knows the distribution parameter of banks’ private borrowing costs,
there is no need for a survey. Therefore, a survey mechanism that induces a distribution-free
expected benchmark bias is desirable.

The major finding of this study is that the expected benchmark bias is distribution-free
under a \textit{standard} survey, that is, agents have independent signals, follow strictly increasing
equilibrium strategies, and have a symmetric payoff function that includes a penalty pro-
portional to his sensitivity to the index and quadratic to his reporting error. This suggests
the administrator can obtain an unbiased index by adjusting it with a number that depends
only on the parameters of the fixing mechanism, the penalty function, and the participants’
weighting on the order statistics of the reports.

Next, I focus on a particular case in which the benchmark equals to the trimmed av-
erage of survey results, and agents’ payoffs depend on the benchmark level. A closed-form
symmetric Bayesian Nash equilibrium strategy exists, and the reporting error depends on
the administrator’s penalty function, the agent’s sensitivity to the benchmark rate, and the
location of the agent’s type in the distribution. The closed-form solution suggests that in-
creasing the size of the panel and trimming more survey responses reduces the expected
benchmark bias, while averaging randomly selected submissions outside the trimmed range
does not. Colluding banks not only manipulate LIBOR more aggressively and effectively, but
also cause the expected benchmark bias no longer distribution-free.

\(^7\)See classical mechanism design papers such as d’Aspremont and Gérard-Varet (1979). Further, if the
private signals are correlated and the correlation is a common knowledge, then the benchmark administrator
can even achieve full-information results as in Cremer and McLean (1988), McAfee, McMillan, and Reny
This paper extends the literature of game of incomplete information by releasing the assumption that principal knows the private signal distribution. Although such assumption is realistic in other games of incomplete information such as auctions, it is inconsistent with the nature of survey. The paper shows that if the agents in the survey are symmetric with i.i.d. private types, then the expected benchmark bias is distribution-free as long as principal implement the penalty function properly.

This paper also speaks to a growing empirical literature studying benchmark manipulation, especially on LIBOR.\textsuperscript{8} The distribution-free expected benchmark bias may serve as a basis to the study of historical benchmark bias. In addition, this study is the first to prove that increasing the panel size while retaining the share of trimmed quotes still reduces the LIBOR bias, while Diehl (2013) evaluates and compares the performance of different aggregators, such as the median, and trimmed average.

Finally, albeit motivated by the LIBOR and ISDAfix scandals, this paper does not intend to offer a complete practical solution to reform a specific benchmark. In addition to Duffie and Stein (2015), which discusses transaction-based LIBOR, Duffie and Dworczak (2014) studies the theoretical foundation of robust and optimal benchmarking under sparse transactions. Duffie, Dworczak, and Zhu (2014) study the price-transparency role of benchmarks in a search market. Coulter and Shapiro (2014) proposes a sequential implementation of the LIBOR mechanism in which a bank’s borrowing cost is observable to the counter-party and at least one other panel bank, following the spirit of Moore and Repullo (1988).

\textsuperscript{8}Specifically, Hou and Skeie (2014) provide a summary of the LIBOR scandal. Hartheiser and Spieser (2010) look at panel banks' submissions and estimate the clustering of LIBOR submissions. Snider and Youle (2010), and Snider and Youle (2012) study the relationship between LIBOR submissions and banks’ credit default swap (CDS) spreads, in various currencies. Abrantes-Metz, Kraten, Metz, and Seow (2011) compare individual bank quotes to CDS spreads and market capitalization data from early 2007 to mid-2008. They find the data are inconsistent with a material manipulation of the US dollar 1-month LIBOR rate, although anomalies in individual quotes do exist. Kuo, Skeie, and Vickery (2012) show that during the financial crisis, banks were willing to borrow from the Fed at a higher interest rate on collateral than the interbank lending rate that the banks reported in the LIBOR fixing process. King and Lewis (2015) study the link between bank CDS spreads to the LIBOR submissions. Gandhi, Golez, Jackwerth, and Plazzi (2016) show that manipulation was initially stronger for banks incorporated outside the U.S., where enforcement is historically weaker, and that it disappeared in the aftermath of LIBOR investigations. Youle (2014) implicitly estimates the positions from the submissions: the result suggests LIBOR distortion that is consistent with Kuo et al. (2012) and Kuo, Skeie, Vickery, and Youle (2014). Eisl, Jankowitsch, and Subrahmanyan (2014) choose not to model the incentive directly but focus on the quantification of the potential effects of manipulated individual contribution to the final rate.
The rest of this paper proceeds as follows. Section 2 presents the survey game and proves the existence of equilibrium under an asymmetric setup. Section 3 defines the standard survey and proves the distribution-free property. Section 4 explores the specific case of trimmed average benchmark and discusses how colluding agents affect a standard survey’s equilibrium outcome. Section 5 summarizes the paper and presents future research opportunities. The appendix contains all proofs and a discussion about signaling in survey.

2 The Model

2.1 Structure and Notation

I consider a game of incomplete information called survey, between a benchmark administrator and a survey panel that contains $N$ risk neutral asymmetric agents, $i = 1, \ldots, N$. Each agent first observes his private signal $s_i \in S_i \equiv [s_i, \pi_i] \subset \mathbb{R}$ and then reports $b_i$, from a compact set $B_i \subset S_i$, to the administrator. Let $S = \prod S_i$ and $B = \prod B_i$. The private signals $s = (s_1, \ldots, s_N)$ can be viewed as actual borrowing costs in the LIBOR fixing mechanism, or true swap quotes in ISDAfix, while $b = (b_1, \ldots, b_N)$ can be viewed as banks’ reports. The density function of $s$ is $f(\cdot)$, and the conditional density of $s_{-i} = (\ldots, s_{i-1}, s_{i+1}, \ldots)$ given $s_i$ is $f_{-i}(s_{-i}|s_i)$, with respect to Lebesgue measure. Both $f(\cdot)$ and $f_{-i}(s_{-i}|s_i)$ are bounded and atomless. The signals $s$ are affiliated so that for all $s', s'' \in S$, $f(s' \lor s'') f(s' \land s'') \geq f(s') f(s'')$. Affiliated $s$ covers the cases in which signals are positively correlated or independent. The distribution of $s$ is a common knowledge only among agents.

The benchmark administrator tries to obtain a benchmark $L$ as a robust estimator of the central moment of private signals $s$. She does not observe the realization of $s$ unless through a costly investigation. Nor does she know the density function $f(s)$, otherwise she can simply calculate the central moment. As a result, the administrator chooses to survey

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9 Throughout the paper, I will use he/his to refer to the participant/agent/bank and she/her to the benchmark administrator/regulator. No association of the roles to particular genders is intended. Also, I will use benchmark and index interchangeably.

10 The operators $\lor$ and $\land$ are component-wise maximum and minimum respectively.
agents and calculate the index $L$ base on reports $b$. Hence,

$$L \equiv \sum_{j=1}^{N} \lambda_j b_j^{(N)},$$  \hspace{1cm} (1)

where $b_j^{(N)}$ is the $j$th order statistics of $b$, for example $b_1^{(N)} = \min (b), \ldots, b_N^{(N)} = \max (b)$, and the weight $\lambda = \{\lambda_1, \ldots, \lambda_N\}$ are non-negative, $\sum_{j=1}^{N} \lambda_j = 1$. Usually the administrator chooses the weight $\lambda$ to reduce the impact of outliers. For instance, both LIBOR and ISDAfix benchmark administrators use $\lambda_j = \frac{1}{N-2n}$ for $n < j \leq N - n$, and $\lambda_j = 0$ for all other $j$, where $0 < n < N$.

In this study, I look at agents’ reporting errors, which is the difference between the report $b$ and signal $s$, under the game of survey. Given a weight $\lambda$, agents’ reporting errors affect the benchmark $L$ by driving it away from its full-information level $L_0$ defined as

$$L_0 \equiv \sum_{j=1}^{N} \lambda_j s_j^{(N)},$$  \hspace{1cm} (2)

under a similar notation for the $j$th order statistics of $s$ as $s_j^{(N)}$. I define the expected difference between $L$ and $L_0$, that is,

$$\mathbb{E} [L - L_0] \equiv \int_S \left( \sum_{i=1}^{N} \lambda_i b_i^{(N)} - \sum_{j=1}^{N} \lambda_j s_j^{(N)} \right) f(s) ds,$$  \hspace{1cm} (3)

as the expected benchmark bias.\textsuperscript{11}

\textbf{Income from benchmark manipulation} Agents may profit from strategical reporting. I focus on the case in which all agents prefer to move the index towards the same direction. Without loss of generality, I let the agents prefer a lower index. Although it is ideal to model arbitrary preference of agents, assuming agents prefer a lower index is consistent with the empirical findings in Snider and Youle (2010), Kuo et al. (2012), and Youle (2014), in the

\textsuperscript{11}The difference between the robust average of $s$, $L_0 \equiv \sum_{j=1}^{N} \lambda_j s_j^{(N)}$, and the central moment of $s$ is outside the scope of this study, which focus on the survey game with incomplete information. More details can be found in the robust statistics literature such as Huber (2011).
specific LIBOR benchmarking case.

Banks prefer a lower LIBOR for two reasons. First, both Grove (1974) and Samuelson (1945), as well as the common sense that banks borrow short-term and lend long-term, suggest that banks usually have maturity mismatch on their balance sheets due to their maturity transformation function. Hence, interest rate changes, in particular for changes in short-term rates benchmarked by LIBOR, affect the net worth of financial institutions. As documented by Akella and Greenbaum (1992), and Lynge and Zumwalt (1980), a 1 basis point increase in the short-term interest rate leads to about 7.7 basis point drop in the bank stock return. The financial reports of publicly traded bank holding companies depict a similar picture. For instance, according to the 10-K forms of the Bank of America, a drop at the short-end of the yield curve helps to boost the bank’s net income. If the short-term yield dropped 100 basis points, Bank of America’s net income could have increased by 536 million US dollars, which is about 2.5% of its net income in 2006, and by 1.255 billion US dollars, which is about 31.3% of its net income in 2008. The sensitivity of net income to the LIBOR index can be considered a size measure of the maturity mismatch of the bank’s balance sheet: a bigger bank with higher maturity mismatch gains more from rigging LIBOR downwards. In addition, the large number of various assets and liabilities on the balance sheet suggests that the bank’s value is locally linear to the index, which is a weighted sum of order statistics. Recent empirical work by Gandhi et al. (2016) shows a positive relation between the end-of-month Libor positions of banks and their submissions.

The second factor is the systemic risk. The financial system faces a difficult choice between financial stability and market transparency in times of systemic banking crisis. Amid the 2008 financial crisis, managers at Barclays believed they were operating under instruction from the Bank of England to lower their LIBOR reports. Whether the Bank of England instructed

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12Flannery and James (1984) find that the change in market rate does not significantly change the return, arguing that the banks have hedged interest rate exposure. However, as the follow-up work of Akella and Greenbaum (1992) shows, Flannery and James (1984) focus mainly on the difference between short-term nominal assets and liabilities, and do not explicitly consider the durations of long-term assets and liabilities.


Barclays—who consistently reported higher cost than others—to lower its submissions or not might never be known for sure. Nevertheless, the major banks had sufficient motive for setting a lower LIBOR, since a higher LIBOR rate would have made the Bank of England’s task of saving the troubled banks, which include not only Barclays but also HBOS and RBS, more difficult. As Morrison and White (2013) show with regard to reputational contagion, one bank’s failure might undermine confidence in the financial system, and cause depositors to withdraw funds from otherwise unconnected banks.

In a more general setup of the survey, I let the agent \(i\)'s income be proportional to a weighted sum of the order statistics of \(b\), that is, \(-\delta_i \sum_{j=1}^{N} w_i^j b_j^{(N)}\), with the non-negative weights \(w_i^j = \{w_i^1, \ldots, w_i^N]\) where \(\sum_{j=1}^{N} w_i^j = 1\). The weights \(w_i^j\) do not have to be the same as the weights \(\lambda\). The negative sign in front of the sensitivity \(\delta_i > 0\) means agents gain from a lower index and suffer from reporting high numbers, without loss of generality. I assume the benchmark administrator can observe \(w_i^j\) and \(\delta_i\) when she audits the agent. In the LIBOR fixing case, the administrator can either obtain precise information through investigation or periodical financial reports, or estimate the sensitivity as in Gandhi et al. (2016). For ISDAfix, the agent’s payoff can be determined by the specifics of derivative positions, which in turn can be verified through an audit process.

**Cost of benchmark manipulation** Agents know that their responses are subject to oversight. For example, the Commodity Futures Trading Commission decides that manipulation of LIBOR breaches Sections 6(c), 6(d), and 9(a)(2) of the Commodity Exchange Act. The Financial Services Authority (FSA), the financial regulatory authority in the U.K., also requires financial institutions to monitor the LIBOR submissions process.\(^\text{15}\) Bankers had been aware of the increased risk when their reports deviated further from the true private value before

\(^\text{15}\)These requirements are derived from the FSA Handbook, which forms part of the FSA rules under the Financial Services and Markets Act 2000, in particular, principles 2, 3, and 5 of the Principles for Businesses: “A firm must conduct its business with due skill, care and diligence[it] must take reasonable care to organise and control its affairs responsibly and effectively, with adequate risk management systems and [it] must observe proper standards of market conduct.”

their scandals unfolded.\textsuperscript{16} Furthermore, recent regulatory activities confirm that banks that gain more from LIBOR rigging incur higher penalties.\textsuperscript{17}

I let agent \(i\)'s expected cost of strategic reporting be a bounded function \(\gamma_i(s_i, b_i) \geq 0\), and the equal sign holds if and only if \(b_i = s_i\). Furthermore, \(\gamma_i(s_i, b_i)\) is a continuous and concave \(C^2\) function such that \(\partial^2 \gamma_i(s_i, b_i)/\partial b_i^2 > 0\), and \(\partial^2 \gamma_i(s_i, b_i)/\partial s_i \partial b_i < 0\). Together with \(\gamma_i(s_i, b_i) = 0\) if and only if \(b_i = s_i\), \(\partial^2 \gamma_i(s_i, b_i)/\partial b_i^2 > 0\) suggests that the penalty increases with the report when the agent sends in a number higher than the truth, that is, \(\partial \gamma_i(s_i, b_i)/\partial b_i > 0\) if and only if \(b_i > s_i\), and decreases with the report otherwise.

**Summary** The income and cost of benchmark manipulation lead to the payoff function of the agent, \(u_i(s_i, b_i) : S_i \times \mathcal{B} \rightarrow \mathbb{R}\), to be

\[
    u_i(s_i, b_i) \equiv -\gamma_i(s_i, b_i) - \delta_i \sum_{j=1}^{N} w_{ij}^b(N).
\]

Following the convention in the literature, the payoff function can also be written as \(u_i(s_i, b_i, b_{-i})\).

Let agent \(j \neq i\) choose \(b_j\) following strategy \(\beta_j : S_j \rightarrow B_j\) with his private signal \(s_j\). I define the objective function of agent \(i\), \(U_i : S_i \times B_i \rightarrow \mathbb{R}\), as

\[
    U_i(s_i, b_i; \beta_{-i}(s_{-i})) \equiv \int_{S_{-i}} u_i(s_i, b_i, \beta_{-i}(s_{-i})) f_{-i}(s_{-i}) \, ds_{-i},
\]

in which \(\beta_{-i}(s_{-i}) \equiv (\ldots, \beta_{i-1}(s_{i-1}), \beta_{i+1}(s_{i+1}), \ldots)\), \(f_{-i}(s_{-i})\) is the joint density function of \(s_{-i} \equiv (\ldots, s_{i-1}, s_{i+1}, \ldots)\), and \(S_{-i}\) is the support of \(s_{-i}\).

\textsuperscript{16}Between January 2005 and September 2009, traders in UBS lobbied the LIBOR submitters for higher and lower rates. Middle managers, who were also actively involved in the manipulation, had one eye on the bank's reputation in an internal e-mail: “as I said before— I dun mind helping on your fixings, but I'm not setting lorum 7bp away from the truth I'll get UBS banned if I do that, no interest in that.” See the article “What the UBS Libor emails said”, BBC, December 19, 2012. Accessed January 20, 2013. http://www.bbc.com/news/business-20781763.

2.2 Agent’s Best Response

I now discuss the agents’ best responses under a pure strategy Bayesian Nash equilibrium (PSBNE).

**Definition.** A pure strategy profile \( \beta(s) = (\beta_1(s_1), \ldots, \beta_N(s_N)) \) is a PSBNE if, for each agent \( i \), \( \beta_i(s_i) \) is the agent’s best response such that \( U_i(s_i, \beta_i(s_i); \beta_{-i}(s_{-i})) \geq U_i(s_i, b'_i; \beta_{-i}(s_{-i})) \) for any other response \( b'_i \).

I do not “normalize” the objective function by subtracting \( U_i(s_i, b_i, b_{-i}) \) with \( U^0_i(s_i) \), where \( U^0_i(s_i) \equiv \int_{S_{-i}} u_i(s_i, s_i, \beta_{-i}(s_{-i})) f_{-i}(s_{-i}) ds_{-i} \) is bank \( i \)'s payoff under truthful reporting. Although the normalization makes the bank’s payoff positive (and therefore, individually rational) under the equilibrium, it does not change the equilibrium choice of bank \( i \) since \( U^0_i(s_i) \) does not depend on the bank’s choice variable \( b_i \).

In reality, a participating agent can also choose to withdraw from the panel. However, although the administrator has limited power to force an outside agent to join the panel, she does have strong power to prevent a participating agent from leaving, especially when the availability of the benchmark is critical to market stability. Therefore, I do not consider the exit option of agents in the definition of the equilibrium.

I then show the existence of a weakly increasing equilibrium in Proposition 1.

**Proposition 1.** A non-decreasing PSBNE \( \beta(s) \) exists.

**Proof.** See Appendix. \( \square \)

The non-decreasing equilibrium strategy suggests that the order among signals is the same as the order among equilibrium bids. Therefore, the expected benchmark bias becomes

\[
\mathbb{E}[L - L_0] = \int_S \sum_{j=1}^{N} \lambda_j \left( \beta \left( s_j^{(N)} \right) - s_j^{(N)} \right) f(s) ds, \tag{5}
\]

\(^{18}\)As reported by Enrich, David in “Banks Warned Not to Leave Libor.” Wall Street Journal. 2013. Accessed November 12, 2013. [http://www.wsj.com/articles/SB10001424127887324432004578302164058534372](http://www.wsj.com/articles/SB10001424127887324432004578302164058534372), During the LIBOR investigation, panel banks that threatened to withdraw—including BNP Paribas and Rabobank—received blunt letters from the FSA warning them not to do so. “...the regulator was simply fulfilling its obligation to ensure the market stability: ‘If one bank goes, all might go’...”
where $\beta_j^{(N)}(s)$ is the $j$th-order statistics of the equilibrium submissions, and $\beta_j^{(N)}(s) = \beta\left(s_j^{(N)}\right)$.

With the existence of equilibrium, I then show a few sample equilibrium best responses in the example below.

**Example.** Consider three surveys, all of which have agents $i = 1, 2, 3$ with independent and identically distributed (i.i.d.) private signals $s_i \in [0, 1]$. The index is calculated as the median of the reports $b = (b_1, b_2, b_3)$, so $L = b_2^{(3)}$. The objective function for agent $i$ is $U_i = \int_{s_{-i}}^{s_i} (- (s_i - b_i)^2 - L) f(s_{-i})ds_{-i} = -(s_i - b_i)^2 - \int_{s_{-i}}^{s_i} b_2^{(3)} f(s_{-i})ds_{-i}$. In the first and second surveys, the private signal follows uniform distribution $f_i(s_i) = 1$ and triangle distribution $f_i(s_i) = 2s_i$, respectively. In the third survey, $s_i$ has a density function $f_i(s_i) = \frac{1}{2\pi} \cos\left(\frac{\pi s_i}{2}\right)$.

From Proposition 1, agent $i$ has an increasing pure strategy equilibrium: this means agent $i$’s report $b_i$ becomes the median (and therefore, $b_i = L$) if and only if agent $i$’s signal $\min(s) \leq s_i \leq \max(s)$, which happens with probability $1 - s_i^2 - (1 - s_i)^2 = 2s_i(1 - s_i)$ in the first survey. Hence the first order condition suggests $0 = \frac{\partial}{\partial b_i} U_i = \frac{\partial}{\partial b_i} (- (s_i - b_i)^2 - 2s_i(1 - s_i)b_i)$, and gives the strategy for agents in the first survey as $\beta(s_i) = s_i^2$. Similarly, the equilibrium strategy for the second survey is $\beta(s_i) = s_i\left(s_i^3 - s_i + 1\right)$, and for the third survey $\beta(s_i) = s_i - \left(1 - \sin\left(\frac{\pi s_i}{2}\right)\right) \sin\left(\frac{\pi s_i}{2}\right)$.

It is intuitive to observe why $\beta_i(s_i) \leq s_i$ under not only all three aforementioned distributions but also more general cases. Since the marginal cost of lying is zero at the true borrowing cost value, truth-telling cannot be an equilibrium. Indeed, consider a small downward deviation: the marginal benefit from reporting a lower number is strictly positive at least for some realization of the signal. Therefore, equilibrium strategies involve some misreporting, and given that reporting a cost strictly above the true value is a strictly dominated strategy, any distortion has to be towards lower values. Section 3 studies the administrator’s option to obtain an unbiased benchmark.
3 Benchmark Administrator’s Problem

The equilibrium result obtained in Section 2 suggest that an agent’s reporting error depends on a few factors: the parameters of the panel $N$ and $n$, the penalty function $\gamma(s_i, b_i)$, and the distribution function of the signals. Among these factors, the density function of $f(s)$ is unknown to the administrator. Hence, a truth-telling mechanism using the revelation principle is not feasible.\footnote{The revelation principle suggests that, with a transfer between the agents and the regulators, we can eliminate the bias induced by the equilibrium reporting strategy. Unfortunately, this usually requires knowledge about the distribution of private signals. Also, although the regulator can and does impose penalties on the misreporting banks, a daily transfer between the regulator and agents is difficult to implement in reality.} As a result, from the regulator’s viewpoint, it is important to design a survey in which the expected benchmark bias does not depend on the signal distribution. I introduce the definition of a standard survey as below.

Definition. A survey is standard if the administrator sets the expected penalty function \[ \gamma(s_i, b_i) = \zeta (s_i - b_i)^2 \geq 0, \] where $\zeta$ is a strictly positive constant and agents have symmetric payoff functions such that $w_i = w_j$ for all $i$, follow strictly increasing equilibrium strategies, and have i.i.d. private signals.

Proposition 2 shows that under a standard survey, although the reporting error $s_i - b_i$ depends on the distribution of $s$, the expected benchmark bias $E[L - L_0]$ is distribution-free.

**Proposition 2.** The expected benchmark bias in a standard survey does not depend on the private signal distribution $f(\cdot)$.

**Proof.** See Appendix. \hfill \Box

Since adding a constant to the payoff function does not change an agent’s optimal strategy $\beta$, Proposition 2 shows that the administrator can adjust the index by adding a number that depends only on the penalty function as well as the parameters of the mechanism, all available to the regulator, but not on the unknown private signal distribution.

**Example.** In this example, I verify Proposition 2, which suggests that all three equilibria with different signal distributions in the example in Section 2 should lead to the same expected
benchmark bias. In the first survey with uniform distribution, the density of the median is 
\( f_2^{(3)} (s_i) = \frac{1}{6} s_i (1 - s_i) \) and the expected benchmark bias becomes

\[
\mathbb{E} \left[ b_2^{(3)} - s_2^{(3)} \right] = \int_0^1 (\beta (s_i) - s_i) f_2^{(3)} (s_i) \, ds_i \\
= \int_0^1 (s_i^2 - s_i) \frac{1}{6} s_i (1 - s_i) \, ds_i \\
= -\frac{1}{5}.
\]

Under the second survey with triangle distribution, in which 
\( f_2^{(3)} (s_i) = \frac{1}{3} s_i^2 (1 - s_i^2) s_i \), the expected benchmark bias is

\[
\int_0^1 (s_i (1 - s_i + s_i^3) - s_i) \frac{1}{3} s_i^2 (1 - s_i^2) s_i \, ds_i = -\frac{1}{5}.
\]

Finally, when \( s_i \in [0, 1] \) has a density function 
\( f_i(s_i) = \frac{1}{2} \pi \cos \left( \frac{\pi s_i}{2} \right) \) for all \( i \), \( f_2^{(3)} (s_i) = 3\pi \cos \left( \frac{\pi s_i}{2} \right) (1 - \sin \left( \frac{\pi s_i}{2} \right)) \sin \left( \frac{\pi s_i}{2} \right) \), and thus, the expected benchmark bias is

\[
\int_0^1 \left( s_i - \left( 1 - \sin \left( \frac{\pi s_i}{2} \right) \right) \sin \left( \frac{\pi s_i}{2} \right) - s_i \right) \\
3\pi \cos \left( \frac{\pi s_i}{2} \right) (1 - \sin \left( \frac{\pi s_i}{2} \right)) \sin \left( \frac{\pi s_i}{2} \right) \, ds_i = -\frac{1}{5}.
\]

Therefore, the administrator can restore the proper level of index in expectations by adding \( \frac{1}{5} \) to the index without worrying about whether the distribution is uniform, triangular, or any other absolutely continuous distribution.

In reality, the weights \( w \) might be hard to measure precisely, which makes it more difficult for the benchmark administrator to adjust the index and recover the unbiased benchmark. However, under the standard survey, the expected benchmark bias remains the same when the cross-sectional distribution of private signals varies over time, as long as \( w \) is stable.

In addition, payoffs of many financial contracts are determined by the change in the benchmark index over a period, rather than the absolute level. For example, Eurodollar futures prices are determined by the market’s expectation of the 3-month US dollar LIBOR
interest rate. The expected payoff of an Eurodollar trader is not affected by the existence of LIBOR bias as long as the bias remains the same over time.20

Implementing the penalty function In general, the administrator implements the penalty function in a two-step process. She first audits the survey participants, and then imposes a fine in case the audit has discovered any irregularities.

The administrator usually audit an agent with a probability: auditing all agents in each period can be prohibitively costly. If the audit reveals that an agent was misreporting, regulators would impose a penalty according to the agent’s size of LIBOR position, which can also be obtained through the audit as the cases of Barclays and UBS suggest.21

It follows that the administrator has great flexibility in setting the audit lottery and ex post fines. Although sparse, interbank lending transactions can provide a good signal about what the LIBOR should be. Other transactions, such as the CDS spread, Term Auction Facilities, or even interbank lending transactions that have occurred before, offer noisy estimates of the real interbank borrowing cost today. Suppose the regulator observes a noisy but unbiased estimation of the agent’s type $\hat{s}_i = s_i + \varepsilon_i$, where $\varepsilon_i$ is independent to $s_i$ and $b_i$. The regulator can audit each agent with probability $\alpha \cdot |\hat{s}_i - b_i|$, and impose a fine $\frac{\zeta \delta_i}{\alpha} \cdot |s_i - b_i|$ after the audit reveals $\delta_i$ and $s_i$.22 In addition, the administrator can audit each agent with a fixed probability $\alpha$ and impose a penalty $\frac{\zeta \delta_i}{\alpha} (s_i - b_i)^2$ after she obtains the $\delta_i$ and $s_i$ from the audit. Both approaches constitute quadratic expected penalties.

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20A Eurodollar price quote of $98.00 implies the market expect the 3-month LIBOR at the settlement date to be $100.00 - $98.00, which corresponds to a 2% LIBOR rate. An Eurodollar trader, who buys one contract when the expected LIBOR rate is 2% and sells the contract when the expected LIBOR rate is 1%, obtains an $1 profit which is not affected by the LIBOR bias as long as the bias remains stable.


22It is a common practice among regulators to investigate selectively according to certain triggers, when monitoring is costly. One prominent example is the tax audit from IRS. In the fiscal year of 2008, IRS audited about 1% of 137 million tax returns, while certain audit triggers boost the chance of winning the audit lottery.
4 Trimmed Average Benchmark: A Special Case with Closed-form Equilibrium

The previous section shows the distribution-free property under a standard survey with arbitrary $\lambda$ and $w$. In many practical surveys, an agent’s payoff links to the benchmark only, rather than each survey responses, so $\lambda = w$. Further, the benchmark is calculated as a trimmed mean to eliminate the top and bottom $n$ outliers. The fixing of ISDAfix follows this practice: the swap cash settlement price is calculated as the interquartile average of the reports from $N = 16$ participating banks: therefore, $w_j = \lambda_j = \frac{1}{N-2n}$ for $n < j \leq N - n$, and $\lambda_j = 0$ for all other $j$, where $n = 4$. An agent’s payoff on his swap position only depends on the fixed swap cash settlement quote, not on other agents’ reports given the settlement price. A bank that manipulates LIBOR only to benefit from its interest-rate related position faces the same situation.

In this section, I focus on a standard survey game with trimmed average benchmark, which has a closed-form equilibrium. The weight $\lambda_j$ is $\frac{1}{N-2n}$ for $n < j \leq N - n$ or 0 otherwise $= \frac{1}{N-2n}$. Without loss of generality, all agents still prefer a lower benchmark, but might have different magnitude of sensitivity $\delta_i$. However, since $\zeta$ is a choice variable of the administrator, she can also scale the penalty by $\delta_i$, as discussed in Section 3. Without loss of generality I can drop $\delta_i$ so that the payoff function becomes

$$u_i(s_i, b) = -\zeta (s_i - b_i)^2 - \sum_{j=1}^{N} \lambda_j b_j^{(N)}.$$  \hspace{1cm} (6)

Proposition 3 characterizes the equilibrium strategy, when the model parameters guarantee the equilibrium strategy $\beta(s_i)$ to be strictly increasing over support $S$. Using similar notation, I note the probability density function of the $n$th order statistics out of $N$ signals as $f^{(N)}_n (\cdot)$.

**Proposition 3.** Given the current index calculation method under which the weight $\lambda_j$ is
\begin{align*}
\frac{1}{N-2n} & \text{ for } n < j \leq N - n \text{ or } 0 \text{ otherwise, agent } i \text{'s PSBNE } \beta : S \to B \text{ satisfies} \\
\beta (s_i) &= s_i - \frac{\Delta (s_i)}{2\zeta(N-2n)}
\end{align*}

where \( \Delta (s_i) \) is

\begin{align*}
\Delta (s_i) &= \frac{(N-1)!}{(n-1)! (N-n-1)!} \int_0^{F(s_i)} \left[ \frac{(1-y)^{N-2n} - y^{N-2n}}{[y(1-y)]^{1-n}} \right] dy,
\end{align*}

when the probability density function of the \( n \)th order statistics \( f_n^{(N-1)} (\cdot) \) and \( N-n \)th order statistics \( f_{N-n}^{(N-1)} (\cdot) \) satisfies the condition for strictly increasing equilibrium

\begin{align*}
\max_{x \in S} \frac{f_n^{(N-1)} (x) - f_{N-n}^{(N-1)} (x)}{N-2n} < 2\zeta. \tag{7}
\end{align*}

Proof. See Appendix.

Figure 1 plots the strictly increasing equilibrium. There are a few things to note. First, \( \Delta (s_i) \) is the probability that agent \( i \)’s equilibrium submission is used in the index calculation, where \( s_i - \frac{1}{2\zeta(N-2n)} \) characterizes the “first-best” report of the agent if he knows his submission will be used. Second, \( F(s_i) \in [0,1] \) suggests that the integral \( \int_0^{F(s_i)} \frac{(1-y)^{N-2n} - y^{N-2n}}{[y(1-y)]^{1-n}} dy \) is non-negative, hence in equilibrium an agent always reports \( \beta (s_i) \leq s_i \). Finally, when an agent has the lowest possible borrowing cost, he reports \( \beta (s_i) = s_i \).

Next, I evaluate how regulation shapes an agent’s submission strategy.

Corollary. The reporting error increases when the regulatory system gets loose, that is, \( \zeta \) gets smaller. If the regulator does not monitor index fixing, then agents will report the lowest possible type.

The equilibrium suggests that the administrator plays an important role. When the regulatory system is not tight, agents are more likely to manipulate the benchmark more aggressively. In the extreme case in which there is no regulatory control, agents will push the benchmark to the lowest level—thus, the index contains no information about the private
This figure shows the agent’s equilibrium submission strategy $\beta(s)$ when the panel contains $N = 16$ agents with i.i.d. types uniformly distributed on [0%, 10%]. The lowest and highest $n = 4$ bids are trimmed. The penalty is $\gamma(s_i, b_i) = 0.05 \cdot (s_i - b_i)^2$. The interest rate sensitivity for agents is normalized to $\delta = 1$. The upper subplot shows the equilibrium strategy $\beta(s)$ as a function of type $s_i$. The lower subplot shows the distribution of types and equilibrium submissions. The optimal strategy causes the submissions of agents to cluster around the lower half of the support.
types at all.

4.1 Comparative Statics

Since the LIBOR manipulation scandal unfolded, there has been much discussion about possible options for strengthening the benchmark fixing mechanism. For instance, Wheatley (2012) considers three options: increase the number of participating agents, increase the number of trimmed reports $n$, or calculate the benchmark by taking the average of a few randomly selected submissions from the central quartiles. In this subsection, I first discuss the comparative statistics of the equilibrium result in Proposition 3, which comment on the options proposed in Wheatley (2012).

Figure 2 addresses the first and the second options under a standard survey in which the support of private signal is [0%, 10%], the penalty function as $\gamma(s_i, b_i) = 0.05 \cdot (s_i - b_i)^2$. The figure shows the expected benchmark bias under different panel sizes $N$ from 16 to 48 and different fractions of trimmed bids $n/N$. When the fraction of bids trimmed remains unchanged, a larger panel (higher $N$) leads to reduced size of expected bias. Is this property true under all possible combination of parameters? Proposition 4 confirms this conjecture.

**Proposition 4.** Increasing both the size of the panel $N$ and the number of trimmed reports, and keeping the proportion $n/N$ the same reduces the reporting error. In other words, note the equilibrium strategy under panel size $N$ and trimmed quotes $n$ as $\beta_{N,n}(s_i)$, for positive rational number $m$ such that $mn$ and $mN$ are integers, $\beta_{N,n}(s_i) \leq \beta_{mN,mn}(s_i)$.

Proposition 5 shows that given the size of the panel $N$, the magnitude of expected benchmark bias decreases consistently when $n$ increases.

**Proposition 5.** Increasing the size of panel $N$ and the number of trimmed reports, and keeping the included reports to be the same reduces the reporting error, so that $\beta_{N,n}(s_i) \leq \beta_{N+2m,n+m}(s_i)$ for positive rational number $m$.

Indeed, Propositions 4 and 5 hold for not only the quadratic penalty function, but also for all penalty functions that can be written as $\gamma(s_i, b_i) = \delta_i \hat{\gamma}(s_i, b_i)$, as shown in Appendix.
This figure shows the expected LIBOR bias with $N$ agents, each with $s_i \in [0\%,10\%]$. The lowest and highest $n$ bids are trimmed. The penalty is $\gamma(s_i, b_i) = 0.05 \cdot (s_i - b_i)^2$. The interest rate sensitivity for agents is normalized to $\delta = 1$. The Y-axis is the expected LIBOR bias in percentages. The X-axis is the proportion of trimmed bids. For example, the point at $n/N = 0.25$ on the $N = 16$ curve shows the expected LIBOR bias when $N = 16$ and $n = 4$. 
The last option, taking the average of randomly selected submissions from the central quartiles, might not work at all. Suppose that each agent in the central quartile has a probability $q$ of being selected. This reduces each agent’s probability of changing the index by a factor of $q$. On the other hand, if agent $i$ is selected in the averaging, his submission now has $1/q$ times more weight. Therefore, the agent’s objective function $U_i(s_i, b)$ is unchanged, as is the equilibrium.

In the following subsections, I discuss non-standard surveys with trimmed average benchmark, in particular, the cases in which agents signal using their submissions or collude.

### 4.2 Collusion

This subsection studies the collusion in survey with trimmed average benchmark and examines whether the distribution-free property still holds. I assume that only a subset of agents forms a collusion ring: First, a larger ring is more likely to fail. Second, the trimming mechanism suggests that it is not necessary to have a ring with more than $N - n$ agents. The agent with the weakly highest sensitivity to the benchmark acts as the leader and calls upon other $m - 1$ agents to collude. Figure 3 illustrates the survey with colluding agents.

Without loss of generality I allow the agent 1 to be the leader. The incentive for participation in the collusion was based on a “scratch your back” basis: therefore, the non-leading agents participate in the collusion ring in the hope that they will be able to become the leader later when their sensitivity to benchmark becomes higher. Therefore, I limit my study to cases without side payments among colluding banks.

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Trader: Alright, well make sure he [the UBS trader] knows
Broker: Yeah, he will know mate. Definitely, definitely, definitely
Submitter: You know, scratch my back yeah an all
Broker: Yeah oh definitely, yeah, play the rules.

One trader told other parties:

“Anytime i can return the favour let me know ...”.

Although there is evidence that the external trader paid back his broker, there was little evidence showing that the trader has paid back all the brokers that participated in the collusion.
This figure shows a sample benchmark survey panel with $N = 8$ agents each with private information $s_i$. Among them agent 1, 2, and 3 are colluding, which means that $m = 3$. The agent 1, with a higher incentive $\delta_1 > \max(\delta_2, \delta_3)$, acts as the leader among the colluding agents and decides the submission $\kappa_3(s_1)$ for all three colluding agents. The rest of the panel, unaware of the collusion, submit their individual strategic bids $\beta(s_i)$. 
Without side payments among the colluding agents, the effective collusion strategy is to have all participants submit the same bids, as in McAfee and McMillan (1992). There is no credible way for a colluding agent to signal his private type: each agent has incentive to claim that he suffers severely in order to join the collusion, hence demands a large side payment. I allow the leader to choose a uniform submission $\kappa$ for all $m$ colluding agents within the ring, while $\kappa$ maximizes the leader’s utility given his type $s_i$. Thus, the agents’ reports are $b = \{\kappa_m(s_1), \ldots, \kappa_m(s_1), b_{m+1}, \ldots, b_N\}$. The payoff of the leading agent 1 and colluding banks $i \in \{2, \ldots, m\}$ are

$$u_1(s_1, b) \equiv -\zeta \delta_1 (s_1 - \kappa_m(s_1))^2 - \delta_1 \sum_{j=1}^{N} \lambda_j b_j^{(N)},$$

and

$$u_i(s_i, b) \equiv -\zeta \delta_i (s_i - \kappa_m(s_1))^2 - \delta_i \sum_{j=1}^{N} \lambda_j b_j^{(N)},$$

respectively. The rest of the agents $j \in \{m+1, \ldots, N\}$ are not aware of the existence of the collusion.

**Proposition 6.** If the equilibrium strategy $\beta(\cdot)$ for each non-colluding agent is strictly increasing, the leading agent decides the equilibrium colluding submission $\kappa_m(\cdot)$ for all the $m$ colluding agents, which satisfies

$$\kappa_m(s_i) = s_i - \frac{\Delta(\kappa_m(s_i))}{2\zeta (N - 2m)},$$

where

$$\Delta(\kappa_m(s_i)) = \sum_{j=1}^{m} \sum_{r=n-m+j}^{(N-m)-(n-m)-j} \binom{N - m}{r} F^r (\beta^{-1}(\kappa_m(s_i))) \left[1 - F(\beta^{-1}(\kappa_m(s_i)))\right]^{N-m-r}.$$

In the case when $m = 1$, the equilibrium degenerates into the case with no collusion so $\kappa_1(s_i) = \beta(s_i)$.

**Proof.** See Appendix. \hfill $\square$
Figure 4 shows the collusive equilibrium strategy $\kappa_m$. It shows that the leading agent manipulates the benchmark more aggressively when more agents join the collusion. This is because the leading agent can manipulate the index more effectively, since his efforts are helped by other colluding banks, and thus, the marginal effect of penalty is not as effective at reducing the reporting error.

Collusion makes correcting the benchmark more difficult, since the magnitude of expected benchmark bias is no longer distribution-free, as the following proposition shows.

**Proposition 7.** The equilibrium strategy $\kappa_m(\cdot)$ characterized in Proposition 6 suggests that the excess benchmark bias introduced by collusion does not satisfy the distribution-free property in Proposition 2.

*Proof. See Appendix.*

The collusion can be blocked by not releasing the individual agent’s report: when the leader cannot verify each agent’s submission, it is not individually rational for the non-leading agents in the ring to follow the leader and submit $\kappa \neq \beta(s_i)$. Nevertheless, the benchmark administrator may want to release the individual agents’ submission, especially when agents are major players in the market. As a compromise, the benchmark administrator may choose to release the agents’ submissions with a delay. For instance, in July 2013, following the recommendations set out in Wheatley (2012), the British Bankers’ Association decided that the publication of individual banks’ LIBOR submissions would be embargoed for 3 months. Although this will make collusion more difficult, the leader will still be able to verify 3 months later whether other colluding agents complied.

Instead of discussing collusive bidding in a repeated game settings, I focus on a static setting assuming the non-leading agents in the bidding ring comply with the leader bank’s request. The intuition remains the same under a repeated game setting: when agents are colluding, the leader manipulates LIBOR more aggressively, causing a larger benchmark bias and the expected benchmark bias to be distribution dependent.
Figure 4: Equilibrium collusive strategy under various $m$

This figure shows equilibrium strategies of the leading agent when $m = 1, 2,$ or $3$ agents colluding, which are $\kappa_1(s)$, $\kappa_2(s)$, and $\kappa_3(s)$. The survey contains $N = 16$ agents with i.i.d. private types uniformly distributed on $[0\%, 10\%]$, and the lowest and highest $n = 4$ bids are trimmed. The penalty is $\gamma(s_i, b_i) = 0.1 \cdot (s_i - b_i)^2$. The upper subplot shows the equilibrium strategy $\beta(s)$ as a function of private type $s$. The lower subplot shows the distribution of private types and equilibrium submissions. The optimal strategy causes the submissions of agents to cluster around the lower half of the support. The figure shows that when more agents are involved in the collusion, the leading agent submits a more aggressive strategic bid.
5 Conclusion

Pervasive benchmark rigging highlights that survey-based benchmark fixing is a game of incomplete information among the benchmark administrator and survey participants. By the nature of financial benchmarking, the distribution of private signals is common knowledge among participants but not to the benchmark administrator. I show the existence of equilibrium under a general asymmetric setting. Nevertheless, without knowing the private signal distribution, the administrator cannot implement a direct mechanism. I show that, when the survey is standard, that is, agents have i.i.d. private signal and symmetric payoff that contains a penalty function whose expectation is quadratic to the participant’s reporting error, the administrator can induce an equilibrium such that the expected index bias does not depend on the unknown distribution of private signals. Hence, the administrator can either restore the unbiasedness in the index by adjusting it according to the parameters of survey mechanism, or know that the change in the unadjusted index is an unbiased estimator of change in the average private signals.

Trimmed average benchmark is a particular way of calculating benchmark which are adopted by LIBOR and ISDAfix. A closed-form solution of the equilibrium exists. Comparative statics suggests that when we continue to keep the mid-quartile quotes, expanding the size of the panel as well as the number of the trimming quotes would reduce the reporting error. Using a smaller range for calculating the index also helps. Collusion and signaling increase the bias and eliminates the distribution-free property of expected index bias. However, recent LIBOR mechanism reform has fixed this problem by embargoing individual agents’ reports.

Future research might focus on modeling agents in two dimensions: their private signals and sensitivities to the benchmark.
A Appendix

A.1 Proofs for the Propositions and Lemmas

Proof of the existence of equilibrium  Before proving the Propositions, I first define the single crossing property of incremental returns of an objective function and the single crossing condition for games of incomplete information, as in Athey (2001) and Milgrom and Shannon (1994).

Definition. (Athey (2001); Milgrom and Shannon (1994)) The objective function $U_i(s_i,b_i)$ satisfies the single crossing property of incremental returns in $(s_i,b_i)$ if, for all $s_i < s_i'$ and $b_i < b_i'$, $U_i(s_i,b_i') - U_i(s_i,b_i) \geq (>)0$ implies $U_i(s_i',b_i') - U_i(s_i',b_i) \geq (>)0$.

Definition. (Athey (2001)) The single crossing condition for games of incomplete information is satisfied if for each $i = 1, \ldots, N$, whenever every opponent $j \neq i$ uses a nondecreasing strategy $\beta_j(s_j)$, agent $i$'s objective function $U_i(s_i,b_i;\beta_{-i}(s_{-i}))$ satisfies single crossing properties of incremental returns in $(s_i,b_i)$.

Before I prove the existence of equilibrium, I first verify that the objective function of a bank does exist and has bounded value in Lemma A.1.

Lemma A.1. $\int_{\Omega} u_i(s_i,b_i,\beta_{-i}(s_{-i})) f_{-i}(s_{-i}|s_i) \, ds_{-i}$ exists and is bounded for all convex $\Omega \subseteq S_{-i}$.

Proof. First, since $S_i = [s_i,s_i] \subset \mathbb{R}$ for all $i$, then $S_{-i} = \ldots \times S_{i-1} \times S_{i+1} \times \ldots$ is convex and bounded on $\mathbb{R}^{N-1}$, thus, all convex set $\Omega \subseteq S_{-i}$ is bounded. Next, since both $\gamma_i(s_i,b_i)$ and $b_i \in B_i \subseteq S_i$ are bounded for all $i$, $u_i$ is finite as well. Together with the fact that $f_{-i}(s_{-i}|s_i)$ is a bounded and atomless probability density function, we know that $\int_{\Omega} u_i(s_i,b_i,\beta_{-i}(s_{-i})) f_{-i}(s_{-i}|s_i) \, ds_{-i}$ exists and is finite for all convex $\Omega \subseteq S_{-i}$. □

Lemma A.2 shows that the object function satisfies single crossing properties of incremental returns in $(s_i,b_i)$, which is a critical step that I use later in the proof of equilibrium existence.

Lemma A.2. For each $i = 1, \ldots, N$, whenever every opponent $j \neq i$ uses a nondecreasing strategy $\beta_j(s_j)$, agent $i$'s objective function $U_i(s_i,b_i;\beta_{-i}(s_{-i}))$ satisfies single crossing properties of incremental returns in $(s_i,b_i)$.

Proof. First I show $u_i(s_i,b)$ is supermodular in $(s_i,b_k)$ for all $k$ by two steps: $u_i(s_i,b)$ is supermodular in $(s_i,b_i)$, as well as $u_i(s_i,b)$ is supermodular in $(s_i,b_k)$ for all $k \neq i$.
To observe that \( u_i(s_i, b) \) is supermodular in \((s_i, b_i)\), notice that with \( s'_i > s_i \) and \( b'_i > b_i \),

\[
u_i(s'_i, (b'_i, b_{-i})) - u_i(s'_i, (s_i, b_{-i})) - (u_i(s_i, (b'_i, b_{-i})) - u_i(s_i, (b_i, b_{-i})))
\]

\[= -\gamma_i(s'_i, b'_i) + \gamma_i(s_i, b'_i) - (-\gamma_i(s'_i, b_i)) + \gamma_i(s_i, b_i))
\]

\[= -\int_{s_i}^{s'_i} \int_{b_i}^{b'_i} \frac{\partial^2}{\partial x \partial y} \gamma_i(x, y) dxdy > 0.
\]

To observe that \( u_i(s_i, b) \) is supermodular in \((s_i, b_k)\) for all \( k \neq i \), notice that with \( s'_i > s_i \) and \( b'_k > b_k \),

\[
u_i(s'_i, (b'_k, b_{-k})) - u_i(s'_i, (s_i, b_{-k})) - (u_i(s_i, (b'_k, b_{-k})) - u_i(s_i, (b_k, b_{-k})))
\]

\[= -\gamma_i(s'_i, b_i) + \gamma_i(s_i, b_i)) - (-\gamma_i(s'_i, b_i)) + \gamma_i(s_i, b_i)) = 0.
\]

In summary, \( u_i(s_i, b) \) is supermodular in \((s_i, b_k)\) for all \( k \). Based on the fact (v) at the page 872 in Athey (2001), and similar results in Athey (1998) and Athey (2002), if \( u_i(s_i, b) \) is supermodular in \((s_i, b_k)\) for all \( k \), and \( s \) is affiliated, then \( U_i(s_i, b_i; \beta_{-i}(s_{-i})) = \int_{s_{-i}} U_i(s_i, b_i; \beta_{-i}(s_{-i})) ds_{-i} \) is supermodular in \((s_i, b_i)\). Then based on the fact (i) at the page 871 of Athey (2001), since \( U_i(s_i, b_i; \beta_{-i}(s_{-i})) \), which is a function of \((s_i, b_i)\), is supermodular in \((s_i, b_i)\), then it satisfies SCP-IR in \((b_i; s_i)\). Since this is true for all \( i = 1, \ldots, N \), the single crossing condition is satisfied.

Now I am ready to prove the Proposition 1.

**Proposition 1.** A non-decreasing PSBNE \( \beta(s) \) exists.

**Proof.** Lemma A.1 shows that the assumption A1 in Athey (2001) is satisfied. Lemma A.2 implies that for each \( i = 1, \ldots, N \), whenever every opponent \( j \neq i \) uses a nondecreasing strategy \( \beta_j(s_j) \), agent \( i \)'s objective function \( U_i(s_i, b_i; \beta_{-i}(s_{-i})) \) satisfies single crossing of incremental returns in \((s_i, b_i)\). This is exactly the single crossing condition for games of incomplete information. Together with the fact that for all \( i \), \( u_i(s_i, b_i; b_{-i}) \) is continuous in both \( b_i \) and \( b_{-i} \), the Corollary 2.1 in Athey (2001) suggests that a non-decreasing PSBNE exists.

**Proof of Proposition 2** I first prove Lemma A.3, which will be used in the proof of Proposition 2 as well as others.

**Lemma A.3.** Under the setup in Proposition 2, if agent \( i \) reports \( b_i \), and all other agents submit
their equilibrium reports, so \( b = (\ldots, \beta(s_{i-1}), b_i, \beta(s_{i+1}), \ldots) \), then under any strictly increasing \( \beta(\cdot) \),

\[
\frac{\partial}{\partial b_i} \int_{S_{-i}} b_j^{(N)} f_{-i}(s_{-i}) ds_{-i} = F_{j-1}^{(N-1)}(s_i) - F_{j}^{(N-1)}(s_i),
\]

where \( b_j^{(N)} \) is the \( j \)-th order statistics of all reports \( b \), and \( F_j^{(N-1)}(\cdot) \) is the cumulative density function of \( j \)-th order statistics of \( s_{-i} \).

\textbf{Proof.} The agent \( i \)'s report \( b_i \) can either happen to be the \( j \)-th smallest among \( b \), or lower or higher than \( b_j^{(N)} \), and nothing else. So I can split the set \( S_{-i} \) into three partitions such that

\[
\begin{align*}
\Omega_1 &= \{ s_{-i} : b_i < \beta_{j-1}^{(N-1)}(s_{-i}) \} \\
\Omega_2 &= \{ s_{-i} : \beta_{j}^{(N-1)}(s_{-i}) < b_i \} \\
\Omega_3 &= \{ s_{-i} : \beta_{j-1}^{(N-1)}(s_{-i}) < b_i < \beta_{j}^{(N-1)}(s_{-i}) \},
\end{align*}
\]

where \( \beta_{j}^{(N-1)}(s_{-i}) \) is the \( j \)-th smallest equilibrium report among \( N-1 \) reports from agents other than \( i \). From the strictly increasing equilibrium strategy, we know that

\[
\begin{align*}
\Omega_1 &= \{ s_{-i} : s_i < s_{j-1}^{(N-1)} \} \\
\Omega_2 &= \{ s_{-i} : s_j^{(N-1)} < s_i \} \\
\Omega_3 &= \{ s_{-i} : s_{j-1}^{(N-1)} < s_i < s_j^{(N-1)} \},
\end{align*}
\]

where \( s_j^{(N-1)} \) is the \( j \)-th smallest signal among \( N-1 \) signals of agents other than \( i \). Use the density function of \( j \)-th order statistics of \( s_{-i} \) as \( f_j^{(N-1)}(\cdot) \) and cumulative density function as \( F_j^{(N-1)}(\cdot) \), I have

\[
\int_{S_{-i}} b_j^{(N)} f_{-i}(s_{-i}) ds_{-i} = \sum_{p=1,2,3} \int_{\Omega_p} b_j^{(N)} f_{-i}(s_{-i}) ds_{-i}
\]

\[
= \int_{b_i}^{s_1} x f_{j-1}^{(N-1)}(x) dx + \int_{s_1}^{b_i} x f_{j}^{(N-1)}(x) dx + b_i \left[ F_{j-1}^{(N-1)}(b_i) - F_{j}^{(N-1)}(b_i) \right].
\]

The regularity of distribution function ensures \( \int_{S_{-i}} b_j^{(N)} f_{-i}(s_{-i}) ds_{-i} \) to be differentiable with respect to \( b_i \), and the derivative is just \( F_{j-1}^{(N-1)}(s_i) - F_{j}^{(N-1)}(s_i) \).

\textbf{Proposition 2.} The expected benchmark bias in a standard survey does not depend on the private
Proof. Under the symmetric agents and i.i.d. distribution, using the notation $b_j^{(N)}$ as defined in Lemma A.3, the agent’s objective function is

$$U_i(s_i,b_i;\beta_i(s_{-i})) = \int_{s_{-i}} -\zeta \delta_i (s_i - b_i)^2 - \delta_i \sum_{j=1}^{N} w_j b_j^{(N)} f_{-i}(s_{-i}) ds_{-i}.$$ 

Using Lemma A.3, the first order condition of agent $i$’s objective function $U_i(s_i,b_i;\beta_i(s_{-i}))$ w.r.t. $b_i$ is

$$0 = s_i - b_i - \sum_{j=1}^{N} \frac{w_j}{2\zeta} \left[ F_{j-1}^{(N-1)}(s_i) - F_j^{(N-1)}(s_i) \right],$$

and the second order condition confirms the maximization of objective function.

Together with the property of order statistics, this suggests that the equilibrium strategy $\beta(\cdot)$ is

$$\beta(s_i) = s_i - \sum_{j=1}^{N} \frac{w_j}{2\zeta} \left[ F_{j-1}^{(N-1)}(s_i) - F_j^{(N-1)}(s_i) \right]$$

$$= s_i - \sum_{j=1}^{N} \frac{w_j}{2\zeta} \left[ \frac{\int_{0}^{F(s_i)} t^{n-2} (1-t)^{N-n+1} dt}{B(n-1,N-n+2)} - \frac{\int_{0}^{F(s_i)} t^{n-1} (1-t)^{N-n} dt}{B(n,N-n+1)} \right].$$

Define the error term as

$$\rho(s_i) = \beta(s_i) - s_i$$

$$= -\sum_{j=1}^{N} \frac{w_j}{2\zeta} \left[ \frac{\int_{0}^{F(s_i)} t^{n-2} (1-t)^{N-n+1} dt}{B(n-1,N-n+2)} - \frac{\int_{0}^{F(s_i)} t^{n-1} (1-t)^{N-n} dt}{B(n,N-n+1)} \right],$$

which is a function of $F(s_i)$.

The expected index error is

$$\mathbb{E}[L-L_0]$$

$$= \int_{x_2}^{x_2} \ldots \left[ \int_{x_2}^{x_2} \lambda_1 \rho(x_1) f_1^{(N)}(x_1|x_2^{(N)} < x_2, \ldots, x_N^{(N)} < x_N) dx_1 \right] \ldots + \lambda_N \rho(x_N) f_N^{(N)}(x_N) dx_N.$$ 

Now with the Markov property of order statistics,

$$f_1^{(N)}(x_1|x_2^{(N)} < x_2, \ldots, x_N^{(N)} < x_N) = f_1^{(N)}(x_1|x_2^{(N)} < x_2)$$

$$= \frac{f(x_1)}{1-F(x_2)},$$

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plug in the formula of $\rho(x)$ it is easy to see we only need to check

$$
\int_{x_2}^{x_2} \left[ \int_0^{F(x_1)} t^{n-2} (1-t)^{N-n+1} \, dt \right] f^{(N)}_1 \left( x_1 | s_2^{(N)} < x_2 \right) \, dx_1
$$

$$
\int_{x_2}^{x_2} \left[ \int_0^{F(x_1)} t^{n-2} (1-t)^{N-n+1} \, dt \right] \frac{f(x_1)}{1 - F(x_2)} \, dx_1
$$

$$
\int_{x_2}^{x_2} \left[ \int_0^{F(x_1)} t^{n-2} (1-t)^{N-n+1} \, dt \right] \frac{1}{1 - F(x_2)} dF(x_1),
$$

which is indeed a function of $F(x_2)$. We can write

$$
\left[ \int_{x_2}^{x_2} \lambda_1 \rho(x_1) f^{(N)}_1 \left( x_1 | s_2^{(N)} < x_2 \right) \, dx_1 \right] = G(F(x_2)),
$$

then plug it back in, we get

$$
\mathbb{E}[L - L_0] = \int_{x_2}^{x_2} \left[ G(F(x_2)) + \lambda_2 \rho(x_2) \right] f^{(N)}_2 \left( x_2 | s_2^{(N)} < x_3 \right) \, dx_2 \ldots + \lambda_N \rho(x_N) \right) f^{(N)}_N \left( x_N \right) \, dx_N.
$$

Similarly, $\int_{x_2}^{x_3} \left[ G(F(x_2)) + \lambda_2 \rho(x_2) \right] f^{(N)}_2 \left( x_2 | s_2^{(N)} < x_3 \right) \, dx_2$ is a function of $F(x_3)$, and we can repeatedly use the Markov property of the order statistics and eventually get

$$
\mathbb{E}[L - L_0] = \int_{x_2}^{x_N} \left[ H(F(x_N)) + \lambda_N \rho(x_N) \right] f^{(N)}_N \left( x_N \right) \, dx_N
$$

$$
\int_{x_2}^{x_N} \left[ H(F(x_N)) + \lambda_N \rho(x_N) \right] \frac{F^{N-1}(x_N)}{B(N,1)} f(x_N) \, dx_N
$$

$$
\int_{x_2}^{x_N} \left[ H(F(x_N)) + \lambda_N \rho(x_N) \right] \frac{F^{N-1}(x_N)}{B(N,1)} dF(x_N).
$$

Consider that $\rho(x_N)$ is an integral with upper bound as $F(x_N)$, again I can treat $F(x_N)$ as an integration variable, and the result is a function of $F(\pi)$ and $F(0)$. Since any distribution CDF function $F$ has $F(\pi) = 1$ and $F(0) = 0$, by construction, the functional form of $F$ does not change $\mathbb{E}[L - L_0]$. Hence, the expected LIBOR bias $\mathbb{E}[L - L_0]$ only depends on $\zeta$, $N$, $\{\lambda_i\}$, and $\{w_i\}$.

\textbf{Proof of Proposition 3}

\textbf{Proposition 3.} Given the current index calculation method under which the weight $\lambda_j$ is $\frac{1}{N-2n}$ for
\[ n < j \leq N - n \text{ or } 0 \text{ otherwise}, \text{ agent } i \text{'s PSBNE } \beta : S \to B \text{ satisfies} \]
\[ \beta(s_i) = s_i - \frac{\Delta(s_i)}{2\zeta(N - 2n)} \]

where \( \Delta(s_i) \) is
\[ \Delta(s_i) = \frac{(N - 1)!}{(n - 1)! (N - n - 1)!} \int_0^{F(s_i)} \frac{(1 - y)^{N-2n} - y^{N-2n}}{[y(1-y)]^{1-n}} dy, \]

when the probability density function of the \( n \)th order statistics \( f_n^{(N-1)}(\cdot) \) and \( N - n \)th order statistics \( f_{N-n}^{(N-1)}(\cdot) \) satisfies the condition for strictly increasing equilibrium
\[ \max_{x \in S} \frac{f_n^{(N-1)}(x) - f_{N-n}^{(N-1)}(x)}{N - 2n} < 2\zeta. \tag{7} \]

**Proof.** Use Lemma A.3,
\[ \frac{\partial}{\partial b_i} \sum_{j=1}^{N} \lambda_j \beta_j^{(N)} f_{-i}(s_{-i}) ds_{-i} = \frac{1}{N - 2n} \left[ F_n^{(N-1)}(\beta^{-1}(b_i)) - F_{N-n}^{(N-1)}(\beta^{-1}(b_i)) \right], \]

and clearly it is bounded in \( [0, \frac{1}{N-2n}] \).

The first derivative of the bank \( i \)’s expected payoff with regard to the choice variable \( b_i \) suggests the equilibrium reporting strategy \( \beta s_i \) is
\[ \frac{\partial \gamma(s_i, \beta(s_i))}{\partial \beta(s_i)} = 2\zeta(s_i - \beta(s_i)) = \frac{1}{N - 2n} \left[ F_n^{(N-1)}(s_i) - F_{N-n}^{(N-1)}(s_i) \right], \]

and the condition for strictly increasing \( \beta(s_i) \) is
\[ \max_{x \in S} \delta \left[ \frac{f_n^{(N-1)}(x) - f_{N-n}^{(N-1)}(x)}{N - 2n} \right] < \frac{\partial^2 \gamma(x, \beta(x))}{\partial \beta(x)^2} = 2\zeta. \]

Writing \( \Delta(s_i) = F_n^{(N-1)}(s_i) - F_{N-n}^{(N-1)}(s_i) \) I have
\[ \beta(s_i) = s_i - \frac{\Delta(s_i)}{2\zeta(N - 2n)}, \]
and with the results in order statistics, and note the incomplete Beta function as \( I_x(a, b) = \frac{B(x)}{B(a, b)} \),

\[
\Delta (s_i) = I_{F(s_i)}(n, N - n) - I_{F(s_i)}(N - n, n)
= \frac{(N - 1)!}{(n - 1)! (N - n - 1)!} \int_0^{F(s_i)} \frac{(1 - y)^{N-2n} - y^{N-2n}}{[y(1 - y)]^{1-n}} dy.
\]

**Proof of Proposition 4**

**Proposition 4.** Increasing both the size of the panel \( N \) and the number of trimmed reports, and keeping the proportion \( n/N \) the same reduces the reporting error. In other words, note the equilibrium strategy under panel size \( N \) and trimmed quotes \( n \) as \( \beta_{N,n}(s_i) \), for positive rational number \( m \) such that \( mn \) and \( mN \) are integers, \( \beta_{N,n}(s_i) \leq \beta_{mN,mn}(s_i) \).

**Proof.** Since under the given setup the equilibrium report \( \beta(s_i) < s_i \), the proposition is equivalent to the claim that the ratio

\[
\frac{s_i - \beta_{mN,mn}(s_i)}{s_i - \beta_{N,n}(s_i)} = \frac{B(mN, mN - mn)}{B(n, N - n)} \int_0^{F(s_i)} \frac{t^{mn-1} (1 - t)^{mN-mn-1} - t^{mN-mn-1} (1 - t)^{mn-1}}{[t^{N-n-1} - t^{N-n-1} (1 - t)^{n-1}] dt}
< 1.
\]

I check \( \frac{mB(mn, mN - mn)}{B(n, N - n)} \) first. Use the inequality of Beta function,

\[
\frac{mB(mn, mN - mn)}{B(n, N - n)} = m \left( \frac{(m - 1) n}{(m - 1) n + (m - 1) (N - n)} \right)^{(m-1)n} \left( \frac{(m - 1) (N - n)}{(m - 1) n + (m - 1) (N - n)} \right)^{(m-1)(N-n)}
= m \left[ \left( \frac{n}{N} \right)^n \left( \frac{N - n}{N} \right)^{N-n} \right]^{(m-1)}.
\]

Check the logarithm of the ratio, and use Jensen’s inequality for the logarithm function, I have

\[
\log \left\{ m \left[ \left( \frac{n}{N} \right)^n \left( \frac{N - n}{N} \right)^{N-n} \right]^{(m-1)} \right\} = \log m + (m - 1) N \left[ \frac{n}{N} \log \frac{n}{N} + \frac{N - n}{N} \log \frac{N - n}{N} \right]
< \log m + (m - 1) \log \left[ 1 - \frac{2n(N-n)}{N^2} \right]^N.
\]
Notice that and since \( n(N-n) \) attains its minimum at \( n = 1 \) or \( n = N-1 \), \( 2n(N-n) \leq 2(N-1) < N \), together with the fact that \( \left[ 1 - \frac{1}{N} \right]^N \) is increasing in \( N \) and converges to \( 1/e \), I have

\[
\log m + (m - 1) \log \left[ 1 - \frac{2n(N-n)}{N^2} \right]^N < \log m + (m - 1) \log \left[ 1 - \frac{1}{N} \right]^N \\
\leq \log m + (m - 1) \log \frac{1}{e} \\
= \log m - (m - 1) < 0,
\]

for all \( m \geq 1 \). Therefore, \( \frac{mB(mn,mN-mn)}{B(n,N-n)} < 1 \).

To see whether \( \int_{0}^{F(s_i)} \left[ m_{n-1}(1-t)^{mN-mn-1} - t^{mN-mn-1} (1-t)^{mn-1} \right] dt \) is less than 1, first notice both the numerator and denominator equal to 0 when \( F(s_i) = 0 \) or \( F(s_i) = 1 \). Also,

\[
\frac{\int_{0}^{1-F(s_i)} \left[ m_{n-1}(1-t)^{mN-mn-1} - t^{mN-mn-1} (1-t)^{mn-1} \right] dt}{\int_{0}^{1-F(s_i)} \left[ t^{n-1} (1-t)^N - t^{N-n-1} (1-t)^{n-1} \right] dt} = \frac{\int_{0}^{F(s_i)} \left[ m_{n-1}(1-t)^{mN-mn-1} - t^{mN-mn-1} (1-t)^{mn-1} \right] dt}{\int_{0}^{F(s_i)} \left[ t^{n-1} (1-t)^N - t^{N-n-1} (1-t)^{n-1} \right] dt},
\]

therefore, I only need to check whether the inequality holds for \( F(s_i) \leq \frac{1}{2} \), where \( t^{n-1} (1-t)^{N-n-1} - t^{N-n-1} (1-t)^{n-1} > 0 \).

I can check the ratio of the derivative of numerator and denominator with respect to \( F(s_i) \) instead. Since the case for \( m = 1 \) is trivially true, I focus on the case in which \( m \geq 2 \), and write

\[
\frac{t^{mn-1} (1-t)^{mN-mn-1} - t^{mN-mn-1} (1-t)^{mn-1}}{t^{n-1} (1-t)^N - t^{N-n-1} (1-t)^{n-1}} = \frac{x_1^m - x_2^m}{x_1 - x_2},
\]

where \( x_1 = t^n (1-t)^{N-n} \), and \( x_2 = t^{N-n} (1-t)^n \). Notice that \( x_1 \) attains its maximum when \( t = n/N \), so \( \log x_1 = \log \left( t^n (1-t)^{N-n} \right) \leq \log \left[ \left( \frac{N}{n} \right)^n \left( \frac{N-n}{N} \right)^{N-n} \right] \). Use the similar sequence of inequalities as above, I have \( x_1 < \frac{1}{e} \). Similarly, \( x_2 < \frac{1}{e} \) as well. Since \( dx^m/dx = mx^{m-1} < me^{1-m} < 1 \) for all \( 0 < x < 1/e \), Hence we have \( x_1^m - x_2^m = \int_{x_1}^{x_2} (x^m)' dx < \int_{x_1}^{x_2} dx = x_1 - x_2 \). Therefore

\[
t^{mn-1} (1-t)^{mN-mn-1} - t^{mN-mn-1} (1-t)^{mn-1} < t^{n-1} (1-t)^N - t^{N-n-1} (1-t)^{n-1},
\]

and

\[
\frac{\int_{0}^{F(s_i)} \left[ m_{n-1}(1-t)^{mN-mn-1} - t^{mN-mn-1} (1-t)^{mn-1} \right] dt}{\int_{0}^{F(s_i)} \left[ t^{n-1} (1-t)^N - t^{N-n-1} (1-t)^{n-1} \right] dt} < 1.
\]
Together with the inequality \( \frac{mB(mn,mN-mn)}{B(n,N-n)} < 1 \), I can then prove the proposition. \(\square\)

**Proof of Proposition 5**

**Proposition 5.** Increasing the size of panel \( N \) and the number of trimmed reports, and keeping the included reports to be the same reduces the reporting error, so that \( \beta_{N,n}(s_i) \leq \beta_{N+2n,n+m}(s_i) \) for positive rational number \( m \).

**Proof.** Since under the given setup the equilibrium report \( \beta(s_i) < s_i \), the proposition is equivalent to the claim

\[
\begin{align*}
    s_i - \beta_{N+2,n+1}(s_i) - [s_i - \beta_{N,n}(s_i)] &= \\
    &= \frac{1}{2\zeta(N-2n)} \left\{ I_{F(s_i)}(n+1, N-n+1) - \left[ I_{F(s_i)}(N-n+1, n+1) \right] \right\} \\
    &\quad - \frac{1}{2\zeta(N-2n)} \left\{ I_{F(s_i)}(n, N-n) - \left[ I_{F(s_i)}(N-n, n) \right] \right\} \\
    &< 0.
\end{align*}
\]

Notice that from the properties of incomplete Beta function,

\[
I_x(n+1, N-n+1) - I_x(N-n+1, n+1) - [I_x(n, N-n) - I_x(N-n, n)] = \frac{x^n(1-x)^N}{(N-n)B(n,N-n)} \left( 1 - \frac{N}{n} (1-x) \right) + \frac{x^{N-n}(1-x)^n}{(N-n)B(N-n,n)} \left( 1 - \frac{N}{n} x \right).
\]

When \( x \in [1/2, 1) \), \( x^{N-n}(1-x)^n \geq x^n(1-x)^{N-n} \), together with \( N/n > 2 \),

\[
\begin{align*}
    \frac{x^n(1-x)^N}{(N-n)B(n,N-n)} &\left( 1 - \frac{N}{n} (1-x) \right) + \frac{x^{N-n}(1-x)^n}{(N-n)B(N-n,n)} \left( 1 - \frac{N}{n} x \right) \\
    &< \frac{x^n(1-x)^N}{(N-n)B(n,N-n)} (1 - 2(1-x)) + \frac{x^{N-n}(1-x)^n}{(N-n)B(N-n,n)} (1 - 2x) \\
    &\leq \frac{x^{N-n}(1-x)^n}{(N-n)B(N-n,n)} (1 - 2x + 1 - 2(1-x)) \\
    &= 0,
\end{align*}
\]

and similarly I can show when \( x \in (0, 1/2] \), \( x^{N-n}(1-x)^n < x^n(1-x)^{N-n} \), together with \( N/n > 2 \),

\[
\begin{align*}
    \frac{x^n(1-x)^N}{(N-n)B(n,N-n)} &\left( 1 - \frac{N}{n} (1-x) \right) + \frac{x^{N-n}(1-x)^n}{(N-n)B(N-n,n)} \left( 1 - \frac{N}{n} x \right) < 0.
\end{align*}
\]

Therefore, \( s_i - \beta_{N+2,n+1}(s_i) - [s_i - \beta_{N,n}(s_i)] < 0 \). In other words, increasing the size of the panel
bank and the trimmed reports by one, while calculate the index using the same number of reports strictly lowers reporting bias for all bank $i$'s with $s_i \in (\bar{s}, \overline{\bar{s}})$.

Proof of Proposition 6

**Proposition 6.** If the equilibrium strategy $\beta(\cdot)$ for each non-colluding agent is strictly increasing, the leading agent decides the equilibrium colluding submission $\kappa_m(\cdot)$ for all the $m$ colluding agents, which satisfies

$$\kappa_m(s_i) = s_i - \frac{\Delta(\kappa_m(s_i))}{2\xi(N - 2n)}$$

where

$$\Delta(\kappa_m(s_i)) = \sum_{j=1}^{m} \sum_{r=n-m+j}^{(N-m)-(n-m)-j} \left( N - m \right) P^r \left( \beta^{-1}(\kappa_m(s_i)) \right) \left[ 1 - F \left( \beta^{-1}(\kappa_m(s_i)) \right) \right]^{N-m-r}.$$ 

In the case when $m = 1$, the equilibrium degenerates into the case with no collusion so $\kappa_1(s_i) = \beta(s_i)$.

**Proof.** With a little abuse of notation, I let $s^{(i)} = s^{i,N-m}$ as the $i$th lowest bid from the $N - m$ banks that are not in the ring, instead of the $N$ banks as in previous sections. The banks in the ring submits the same bid $\rho = \beta(k)$, therefore, pretending they are strategically behaved (but without collusion) as type $k$.

$$(N - 2n) E[L] = \sum_{i=n+1}^{N-m-n} \beta \left( s^{(i)} \right) + \sum_{j=1}^{m} \left\{ \int_{k}^{\bar{k}} \beta(x) f_{n-m+j}(x) dx + \int_{k}^{\bar{k}} \beta(x) f_{(N-m)-(n-m)-j+1}(x) dx \right\} + \sum_{j=1}^{m} \beta(k) \left( F_{n-m+j}(k) - F_{(N-m)-(n-m)-j+1}(k) \right),$$

so

$$\frac{\partial}{\partial k} (N - 2n) E[L] = \sum_{j=1}^{m} \beta'(k) \left( F_{n-m+j}(k) - F_{(N-m)-(n-m)-j+1}(k) \right).$$

Hence the optimal $\kappa = \beta(k)$ is that

$$\frac{\partial}{\partial \beta'(k)} \left[ E[L] - E \left[ \gamma (\beta(k) - s)^2 + \xi(\beta(k)) \right] \right] = 0,$$

from which I can get the equilibrium submission $\kappa = \beta(k)$. 

□
Proof of Proposition 7

Proposition 7. The equilibrium strategy $\kappa_m(\cdot)$ characterized in Proposition 6 suggests that the excess benchmark bias introduced by collusion does not satisfy the distribution-free property in Proposition 2.

Proof. Since $\kappa_m(s_i)$ depends on $\beta^{-1}(\cdot)$, and

$$\beta(s_i) = s_i - \frac{1}{2\kappa(N-2n)} \Delta(s_i) \leq s_i,$$

where

$$\Delta(s_i) = \frac{(N-1)!}{(n-1)!(N-n-1)!} \int_0^{F(s_i)} \frac{(1-y)^{N-2n} - y^{N-2n}}{[y(1-y)]^{1-n}} dy,$$

$\kappa_m(s_i) - s_i$ in general cannot be written as a function of $F(s_i)$ only, but also a function of $s_i$. As a result, I cannot treat $F(s_i)$ as an integration variable in the expectation $\int F(s_i) \kappa_m(s_i) dF(s_i)$. Therefore, the integral depends on the functional form of $F(\cdot)$: the expected excess LIBOR bias introduced by collusion is not distribution-free. \qed

A.2 Extensions for the Proposition 4 and 5

Proposition A.4. Proposition 4 and 5 hold for a more general penalty function $\gamma(s_i, b_i) = \delta_i \hat{\gamma}(s_i, b_i)$.

Proof. With general penalty function $\gamma(s_i, b_i) = \delta_i \hat{\gamma}(s_i, b_i)$, I have

$$\frac{\partial \hat{\gamma}(s_i, \beta_{N,n}(s_i))}{\partial \beta_{N,n}(s_i)} = \frac{1}{N-2n} \left[ I_{F(s_i)}(n, N-n) - I_{F(s_i)}(N-n, n) \right]$$

$$\frac{\partial \hat{\gamma}(s_i, \beta_{N+2,n+1}(s_i))}{\partial \beta_{N+2,n+1}(s_i)} = \frac{1}{N-2n} \left[ I_{F(s_i)}(n+1, N-n+1) - I_{F(s_i)}(N-n+1, n+1) \right],$$

and it is easy to see in the proof of Proposition 5,

$$0 > \frac{\partial \hat{\gamma}(s_i, \beta_{N+2,n+1}(s_i))}{\partial \beta_{N+2,n+1}(s_i)} - \frac{\partial \hat{\gamma}(s_i, \beta_{N,n}(s_i))}{\partial \beta_{N,n}(s_i)}$$

$$= \int_{\beta_{N,n}(s_i)}^{\beta_{N+2,n+1}(s_i)} \frac{\partial \hat{\gamma}(s_i, y)}{\partial s_i} dy.$$ 

Notice that, $\frac{\partial \hat{\gamma}(s_i, y)}{\partial s_i} < 0$, hence $\beta_{N,n}(s_i) < \beta_{N+2,n+1}(s_i)$. 

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Similarly,

\[
\frac{\partial \hat{\gamma}(s_i, \beta_{mN,mn}(s_i))}{\partial \beta_{mN,mn}(s_i)} - \frac{\partial \hat{\gamma}(s_i, \beta_{N,n}(s_i))}{\partial \beta_{N,n}(s_i)} = \frac{mN - 2mn I_{F(s_i)}(mn, mN - mn) - I_{F(s_i)}(mN - mn, mn)}{N - 2n} \frac{I_{F(s_i)}(n, N - n) - I_{F(s_i)}(N - n, n)}{
\frac{\partial \hat{\gamma}(s_i, \beta_{mN,mn}(s_i))}{\partial \beta_{mN,mn}(s_i)} \frac{\partial \hat{\gamma}(s_i, \beta_{N,n}(s_i))}{\partial \beta_{N,n}(s_i)}},
\]

and from the proof of Proposition 4,

\[
1 > \frac{\partial \hat{\gamma}(s_i, \beta_{mN,mn}(s_i))}{\partial \beta_{mN,mn}(s_i)} \frac{\partial \hat{\gamma}(s_i, \beta_{N,n}(s_i))}{\partial \beta_{N,n}(s_i)},
\]

hence

\[
0 > \frac{\partial \hat{\gamma}(s_i, \beta_{mN,mn}(s_i))}{\partial \beta_{mN,mn}(s_i)} - \frac{\partial \hat{\gamma}(s_i, \beta_{N,n}(s_i))}{\partial \beta_{N,n}(s_i)} = \int_{\beta_{N,n}(s_i)}^{\beta_{mN,mn}(s_i)} \frac{\partial \hat{\gamma}(s_i, y)}{\partial s_i \partial y} dy.
\]

With \( \frac{\partial \hat{\gamma}(s_i, y)}{\partial s_i \partial y} < 0 \), I have \( \beta_{N,n}(s_i) < \beta_{mN,mn}(s_i) \). Therefore, both Proposition 5 and 4 holds under a more general penalty function \( \gamma(s_i, b_i) = \delta \hat{\gamma}(s_i, b_i) \).

### A.3 Discussion about Signaling

During the LIBOR benchmark fixing process, a bank might reveal his bank specific credit riskiness by reporting a high interbank borrowing cost, in addition to the level of systemic risk. Such a signal might concerns the bank’s short-term funding providers, who may close positions abruptly, leading to a costly fire sales of long-term assets as in Shleifer and Vishny (2011). Hence, banks reported lower than actual borrowing costs when market was concerned about their creditworthiness.

To the best of my knowledge, signaling has only happened in LIBOR fixing so far. Nevertheless, in any survey, agents are able to signal through reporting as long as the public can identify any agent’s report. Hence, it is worthwhile to check whether the distribution-free property of the expected benchmark bias still holds in a standard survey if agents have the will and the power to signal.

Let agent \( i \), by submitting \( b_i \), incurs a signaling cost \( \xi(b_i) \). I assume that \( \xi : S \rightarrow \mathbb{R}^+ \) is a strictly increasing, concave, and \( C^2 \) function that satisfies \( \xi'(b_i) \rightarrow 0 \) as \( b_i \rightarrow 0 \). In addition, I normalize \( \xi(0) = 0 \). I use a normalized quadratic functional form \( \xi(b_i) = \xi b_i^2 \) in the following analysis. Hence,
the agent’s payoff function under signaling becomes $u_i : S \times B \to \mathbb{R}$ as

$$u_i (s_i, b) \equiv -\zeta \delta (s_i - b_i)^2 - \xi b_i^2 - \delta \sum_{j=1}^{N} \lambda_j (s_i - b_i)$$

(8)

where the coefficient $\lambda_j = \frac{1}{N-2n}$ for $n < j \leq N - n$, and $\lambda_j = 0$ for all the other $j$.

Proposition A.5 characterizes the equilibrium submission strategy under signaling.

**Proposition A.5.** If the probability density function of the $n$th-order statistics $f_n^{(N-1)} (\cdot)$ and $N-n$th-order statistics $f_{N-n}^{(N-1)} (\cdot)$ satisfies the condition for strictly increasing equilibrium, then the equilibrium strategy $\beta : S \to B$ for each agent $i$ is strictly increasing and

$$\beta (s_i) = \frac{\zeta \delta s_i}{\zeta \delta + \xi} - \frac{\delta}{2(\zeta \delta + \xi)(N-2n)} \Delta (s_i)$$

where $\Delta (s_i)$ is

$$\Delta (s_i) = \frac{(N-1)!}{(n-1)! (N-n-1)!} \int_{0}^{F(s_i)} \frac{[1-y]^{N-2n} - y^{N-2n}}{[y(1-y)]^{1-n}} dy.$$  

**Proof.** The first order derivative of the payoff function suggests the equilibrium is

$$\beta (s_i) = \frac{\zeta \delta s_i}{\zeta \delta + \xi} - \frac{\delta}{2(\zeta \delta + \xi)(N-2n)} \left[ F_n^{(N-1)} (s_i) - F_{N-n}^{(N-1)} (s_i) \right],$$

where

$$\Delta (s_i) = \frac{(N-1)!}{(n-1)! (N-n-1)!} \int_{0}^{F(s_i)} \frac{[1-y]^{N-2n} - y^{N-2n}}{[y(1-y)]^{1-n}} dy.$$  

A sample equilibrium strategy is plotted in Figure 5.

Signaling causes an agent to report low borrowing costs, and a larger benchmark bias. Signaling has another negative consequence: in general, the expected benchmark bias loses its distribution-free property when agents signal their credit quality in the benchmark fixing process, as shown in Proposition A.6. Therefore it is harder to correct the benchmark when agents have the incentive to signal.

**Proposition A.6.** The equilibrium strategy $\beta (\cdot)$ characterized in Proposition A.5 suggests that the expected excess benchmark bias under signaling does not satisfy the distribution-free property in Propo-
This figure shows the equilibrium strategy when the survey contains $N = 16$ agents with i.i.d. borrowing costs uniformly distributed on $[0\%, 10\%]$. The lowest and highest $n = 4$ bids are trimmed. The penalty is $\gamma(s_i, b_i) = 0.1 \cdot (s_i - b_i)^2$, the signaling parameter $\xi = 0.02$, and the sensitivity is normalized to $\delta = 1$. The upper subplot shows the equilibrium strategy $\beta(s)$ as a function of borrowing cost $s$. The lower subplot shows the distribution of borrowing costs and equilibrium submissions. The signaling causes the submissions of agents to be even lower.
Proof. A similar approach as in the proof of Proposition 7 suggests that the benchmark error under signaling is

\[
E [L - L_0] = \int_\mathbb{R} \left[ \int_\mathbb{R} \lambda_1 \rho(x_1) f_1^{(N)} (x_1 | s_2^{(N)} < x_2, \ldots, s_N^{(N)} < x_N) \, dx_1 \right] \ldots + \lambda_N \rho(x_N) \right] f_N^{(N)} (x_N) \, dx_N,
\]

where

\[
\rho(x_1) = \frac{\xi x_1}{\zeta \delta + \xi} + \frac{\delta}{2 (\zeta \delta + \xi) (N - 2n)} \Delta (x_1)
\]

\[
\ldots
\]

\[
\rho(x_N) = \frac{\xi x_N}{\zeta \delta + \xi} + \frac{\delta}{2 (\zeta \delta + \xi) (N - 2n)} \Delta (x_N).
\]

Although \(\Delta (s_i^{(N)})\) can be represented as a function of \(F(s_i)\), the integration variable \(x_i\) above suggests that the integral contains a term like \(\int_0^\infty x dF(x)\). Therefore, the expectation does depend on the distribution function \(F(\cdot)\).

Similar to collusion, signaling could be fixed easily by limiting access to the agent’s benchmark reports to regulators only. Without the information of agents’ borrowing costs, the market participants would not be able to infer agents’ types. Therefore, the administrator can improve the accuracy of the benchmark by reducing the information available to market.

References


Coulter, B. and J. D. Shapiro (2014). A mechanism for libor. *Available at SSRN 2256952*.


